ELSEVIER

International Journal of Industrial Organization
13 (1995) 195–211

International Journal of Industrial Organization

La Pléiade and exchange rate pass-through

Andrzej Baniak[a,b], Louis Phlips[b,*]

[a]*Oskar Lange Academy of Economics, Komandorska 118/120, 53-345 Wrocław, Poland*
[b]*European University Institute, San Domenico di Fiesole, 50016 Florence, Italy*

Received 22 March 1994; Accepted 13 June 1994

Abstract

We examine the effects of a change in the exchange rate on sales and prices in the framework of a two-country, two-commodity duopoly model with joint production. We distinguish two kinds of reaction. When the firm located in the country whose currency depreciates (appreciates) increases (decreases) sales in both countries, we call it the 'firm-specific' effect. If all sales in the country which appreciates (depreciates) its currency increase (decrease), we call it the 'country-specific' effect. Strategic substitutability, economies of joint production and/or economies of scale lead to the firm-specific effect. Strategic complementarity, diseconomies of joint production and/or diseconomies of scale lead to the country-specific effect.

Keywords: Exchange rate pass-through; Oligopoly; Strategic complementarity; Strategic substitutability

JEL classification: F12; L13

1. Introduction

The model presented in this paper is motivated by the press announcement that the French publishing house Gallimard is putting out an 'Italian Pléiade'. Every cultivated person supposedly knows 'La Pléiade', Gallimard's French high-quality edition of the collected works of famous

* Corresponding author.

SSDI 0167-7187(94)00452-8

novelists. The 'Italian Pléiade' includes Italian translations and competes directly with a collection of comparable quality by the Italian publishing house Mondadori. Gallimard's initiative came at a time when the Italian lira was going through a series of devaluations.

This fait divers served as a guide to our analysis of exchange rate pass-through. There are two markets: the French market (x) and the Italian market (y). In each market there are two sellers: Gallimard (player g) and Mondadori (player m). The two markets are treated separately: to travel from Italy to France or vice versa is expensive for a buyer who wants to buy one of these books, so that prices can differ between markets; in addition, the distribution systems are separated by exclusive dealing.[1]

Mondadori sells its collection in both markets. Gallimard sells its *two* collections in both markets. Suppose, to simplify the analysis, that Mondadori's product is a perfect substitute for Gallimard's Italian Pléiade. Then we have two products on each market: the French Pléiade (product a) and the Italian translations (product b). These two products could be complements (for language teachers and students) or substitutes (for general readers who are fluent in both French and Italian). They could also be independent (for buyers who read only in one of the two languages).

Product a (the French Pléiade) is sold by one producer only (Gallimard) in both markets. However, product b(the Italian translation) is sold by the two producers in both markets.

The question to be answered is: How does a depreciation of the lira affect the sales and the prices of the products in Italy and in France, respectively? We will show that Mondadori will sell more in both countries and that Gallimard, to the contrary, will lose sales of its two products in both markets whether the two products are substitutes, complements or independent (from the consumer's point of view), on one condition: both producers should consider their product(s) as 'strategic substitutes'. Strategic substitutability occurs when it is in a firm's interest to react to an increase in its competitor's sales by a decrease in its own sales and vice versa. On the other hand, when it is in their interest to react to an 'aggressive' sales policy by an increase in sales (this is called 'strategic complementarity'), then both Mondadori and Gallimard will increase sales in France and decrease sales in Italy, whether products a and b are substitutes, complements or independent. Whether a good is considered a strategic substitute or a strategic complement by a player depends on this player's market share and on the curve of the demand function.

Strategic substitutability and strategic complementarity thus play a crucial role. Strategic substitutability leads to increased sales in both countries for

[1] We ignore the occasional arbitrage telephone call from an Italian language teacher to a friend in Paris asking him or her to buy there at a lower price and send by mail.

the firm located in the country whose currency depreciates. We will call this the *firm-specific effect*. Strategic complementarity leads to increased sales in the country that appreciates and to decreased sales in the country that devalues for both firms. We will call this the *country-specific effect*.

The normal price reaction is for prices to decrease in the country that appreciates and to increase in the country that devalues. When the relationships between two commodities (on the demand side and/or the cost side) are taken into account, as we do in our model, other price reactions can appear. We will show that if products a and b are independent on the demand side, the price of a can go *up* in both countries, under certain cost conditions (economies of scale or of joint production for Gallimard) and strategic substitutability. This is a surprising result at first sight. Yet, it is a straightforward consequence of the fact that Gallimard is then a price discriminating monopolist.

Our model uses unspecified demand functions, and is thus more general than the approach followed by Martin and Phlips (1994), who consider a duopoly model with differentiated products but operating in a linear framework. We build on the papers by Hens et al. (1991) and Kirman and Phlips (1992), who use general demand and cost functions but consider only one homogeneous good in each market. The points where we extend their results will be indicated as we proceed.

Section 2 presents the model, introduces the concepts of strategic substitutes and strategic complements and the a priori restrictions imposed for comparative statics. In Section 3 the effects of an appreciation on quantities and prices are analyzed under the assumption that there is no strategic interaction between duopolists. Section 4 allows for economies and diseconomies of joint production. We show that economies of joint production connected with strategic substitutability lead to the firm-specific effect. In Section 5 the effects of economies of scale are analyzed. The proofs of Propositions 2–6 can be found in EUI Working Paper No. 94/20 or can be obtained from the authors upon request.

2. The model

There are two countries (markets), x and y, which are separated in the sense that demands in one country are independent of the prices in the other country. The duopolistic structure is as follows: there is one firm, g, which is located in country x, and one firm, m, located in country y. There are two products, a and b, which can be independent, imperfect substitutes or complements. Firm g produces both products; firm m produces only b. Product b is homogeneous. Both products are sold in both countries. To suppose that only one of the duopolists produces the two goods makes the model tractable without loss of generality.

The inverse demand function for good a in country x is

$$p_{ax} = p_{ax}(x_a, x_b^g + x_b^m)\,, \tag{1}$$

where x_a is the quantity of good a produced by firm g and sold in country x, x_b^g is the quantity of good b produced by firm g and sold in country x, and x_b^m is the quantity of good b produced by firm m and sold in country x. Let $x_b = x_b^g + x_b^m$ be the total quantity of good b sold in country x. We assume that $\partial p_{ax}/\partial x_a = p_{ax}^1 < 0$ and, if goods a and b are substitutes (complements), then $\partial p_{ax}/\partial x_b = p_{ax}^2 < (>)0$.

We can define other inverse demand functions in a similar way:

- for good b in country x:

$$p_{bx} = p_{bx}(x_a, x_b)\,, \tag{2}$$

  where $p_{bx}^2 < 0$ and if goods are substitutes (complements), then $p_{bx}^1 < (>)0$;
- or good a in country y:

$$p_{ay} = p_{ay}(y_a, y_b^g + y_b^m) = p_{ay}(y_a, y_b)\,, \tag{3}$$

  where y_a, y_b^g, and y_b^m are quantities sold in country y, and $y_b = y_b^g + y_b^m$;
- for good b in country y:

$$p_{by} = p_{by}(y_a, y_b)\,. \tag{4}$$

We assume that the inverse demand functions are twice continuously differentiable. Analogous assumptions about the signs of partial derivatives (as in the first two inverse demand functions) are imposed.

The cost function for firm g is

$$c_g = c_g(x_a + y_a, x_b^g + y_b^g)\,; \tag{5}$$

and for firm m:

$$c_m = c_m(x_b^m + y_b^m)\,. \tag{6}$$

The above formulation of the cost functions is somewhat restrictive: it reduces the concept of economies of scope to one of economies of scale.

It is assumed that cost functions are twice continuously differentiable and that marginal costs are positive. We will say that both firms exhibit *economies* (*diseconomies*) *of scale* if

$$c_g^{11}, \quad c_g^{22}, \quad c_m'' < (>)0\,. \tag{7}$$

Firm g exhibits *economies* (*diseconomies*) *of joint production* if the cross-partial derivative, c_g^{12}, is negative (positive).

The exchange rate, e, is defined as the value of the currency of country x (French francs) expressed in the currency of country y (Italian lira).

Thus, firm g earns profits

$$\begin{aligned}\Pi_g &= \Pi_g(x_a, x_b^g, x_b^m, y_a, y_b^g, y_b^m) \\ &= p_{ax}x_a + p_{bx}x_b^g + (1/e)(p_{ay}y_a + p_{by}y_b^g) - c_g \,, \end{aligned} \tag{8}$$

in the currency of country x, while firm m earns

$$\Pi_m = \Pi_m(x_a, x_b^g, x_b^m, y_a, y_b^g, y_b^m) = ep_{bx}x_b^m + p_{by}y_b^m - c_m \,, \tag{9}$$

in the currency of country y.

Thus, there are six first-order conditions that must be satisfied in the interior Nash equilibrium:

$$\Pi_g^1 = p_{ax}^1 x_a + p_{ax} + p_{bx}^1 x_b^g - c_g^1 = 0 \,, \tag{10}$$

$$\Pi_g^2 = p_{ax}^2 x_a + p_{bx} + p_{bx}^2 x_b^g - c_g^2 = 0 \,, \tag{11}$$

$$\Pi_m^3 = e(p_{bx}^2 x_b^m + p_{bx}) - c_m' = 0 \,, \tag{12}$$

$$\Pi_g^4 = (1/e)(p_{ay}^1 y_a + p_{ay} + p_{by}^1 y_b^g) - c_g^1 = 0 \,, \tag{13}$$

$$\Pi_g^5 = (1/e)(p_{ay}^2 y_a + p_{by} + p_{by}^2 y_b^g) - c_g^2 = 0 \,, \tag{14}$$

$$\Pi_m^6 = p_{by}^2 y_b^m + p_{by} - c_m' = 0 \,. \tag{15}$$

The first three equations refer to country x, the last three to country y.

We are interested in how the equilibrium given by the system of equations (10)–(15) reacts to an exogenous exchange rate change. Generally speaking, the increase in e has two direct effects, namely firm g, located in country x whose currency appreciates, decreases sales of its two products in country y: $\mathrm{d}y_a/\mathrm{d}e < 0$, $\mathrm{d}y_b^g/\mathrm{d}e < 0$; and firm m, located in country y which depreciates its currency, increases sales of product b in country x: $\mathrm{d}x_b^m/\mathrm{d}e > 0$.

Then we can distinguish two further effects, namely the changes in sales of both firms on their domestic markets.

A first possibility is that firm m sells more in its domestic market, y, whereas firm g decreases sales in its domestic market, x. In this case, firm m, located in country y which depreciates its currency, sells more of its product in *both* countries:

$$\frac{\mathrm{d}x_b^m}{\mathrm{d}e} > 0, \qquad \frac{\mathrm{d}y_b^m}{\mathrm{d}e} > 0 \,. \tag{16}$$

The second duopolist, firm g, from country x which appreciates its currency, decreases sales of its two products in *both* countries:

$$\frac{\mathrm{d}x_a}{\mathrm{d}e}<0, \qquad \frac{\mathrm{d}x_b^g}{\mathrm{d}e}<0, \qquad \frac{\mathrm{d}y_a}{\mathrm{d}e}<0, \qquad \frac{\mathrm{d}y_b^g}{\mathrm{d}e}<0 . \tag{17}$$

We will call such a response to the change in the exchange rate *the firm-specific effect.*

A second possibility is for firm m to sell less in domestic market y, while firm g increases sales in domestic market x. Then, the sales of *all* products increase in the country which appreciates its currency:

$$\frac{\mathrm{d}x_a}{\mathrm{d}e}>0, \qquad \frac{\mathrm{d}x_b^g}{\mathrm{d}e}>0, \qquad \frac{\mathrm{d}x_b^m}{\mathrm{d}e}>0 , \tag{18}$$

and *all* sales in the country which depreciates its currency decrease:

$$\frac{\mathrm{d}y_a}{\mathrm{d}e}<0, \qquad \frac{\mathrm{d}y_b^g}{\mathrm{d}e}<0, \qquad \frac{\mathrm{d}y_b^m}{\mathrm{d}e}<0 . \tag{19}$$

We will call such a reaction *the country-specific effect.*

The way a change in e affects prices depends on aspects of both countries' market and production structures. To study the problem more thoroughly we introduce the concepts of strategic substitutes and strategic complements.

2.1. Strategic substitutes and strategic complements

This terminology was introduced by Bulow et al. (1985). Strategic substitutes and strategic complements are defined by whether a more 'aggressive' strategy by one duopolist lowers or raises the other duopolist's marginal profits.

Firm g regards commodity b as a *strategic substitute* in market x when

$$\frac{\partial^2 \Pi_g}{\partial x_b^m\, \partial x_b^g} = \Pi_g^{23} < 0 . \tag{20}$$

In other words, an increase in firm m's sales reduces the marginal profitability of firm g. Conversely, firm g regards commodity b as a *strategic complement* when the cross-partial derivative in Eq. (20) is positive.

The concept of strategic substitutes and strategic complements has an interesting interpretation when the duopolists compete in one market. In this case the slope of the reaction function for firm g is given by

$$-\frac{\partial^2 \Pi_g}{\partial x_b^m\, \partial x_b^g}\Bigg/\frac{\partial^2 \Pi_g}{\partial x_b^{g2}} = -\frac{\Pi_g^{23}}{\Pi_g^{22}} . \tag{21}$$

Under the assumption of strict concavity of the profit function, the denominator in Eq. (21) is negative. Hence, if good b is a strategic substitute (complement) for firm g, then the reaction function is downward sloping (upward sloping). We can interpret this in another way. Good b is a strategic substitute (complement) for firm g if the optimal response to a more aggressive strategy (increase in sales) of firm m is to decrease (increase) sales.

In our model we have two commodities in every market. Hence we can distinguish different kinds of strategic substitutability or complementarity. Eq. (20) gives the usual definition. In a differentiated market where goods a and b are imperfect substitutes, we can define a new concept involving the two goods. We will say that firm g considers commodity a as a strategic substitute (complement) to b in market x if

$$\frac{\partial^2 \Pi_g}{\partial x_b^m \, \partial x_a} = \Pi_g^{13} < (>) 0 . \qquad (22)$$

The situation of m is different because it produces only one commodity and faces two possible strategic actions by firm g, namely through a change of quantity in product a or product b.

Thus, we can say in the usual way that firm m regards commodity b as a strategic substitute (complement) in country x if

$$\frac{\partial^2 \Pi_m}{\partial x_b^g \, \partial x_b^m} = \Pi_m^{32} < (>) 0 . \qquad (23)$$

By analogy with (22), we will say that firm m regards commodity b as a strategic substitute (complement) to a in country x if

$$\frac{\partial^2 \Pi_m}{\partial x_a \, \partial x_b^m} = \Pi_m^{31} < (>) 0 . \qquad (24)$$

Similar definitions apply to market y. Note that in a linear framework, if products a and b are substitutes (complements) on the demand side, then they are treated as strategic substitutes (complements) by both producers. Thus, demand and strategic substitutability (complementarity) coincide.

If products are independent, i.e. the price of commodity b does not depend on the sales of commodity a and vice versa, then the inverse demand functions reduce to

$$p_{ax} = p_{ax}(x_a), \qquad p_{bx} = p_{bx}(x_b), \qquad p_{ay} = p_{ay}(y_a), \qquad p_{by} = p_{by}(y_b) , \qquad (25)$$

and $\Pi_m^{31} = \Pi_g^{13} = 0$. In other words, there is no strategic interaction between

a and b. The concept of strategic substitutability and strategic complementarity hence applies only to commodity b since a is produced by g only. In this case a strategic interaction has an interesting interpretation for good b. From Eqs. (11) and (12) we have

$$\Pi_g^{23} = p'_{bx} + p''_{bx} x_b^g \quad \text{and} \quad \Pi_m^{32} = p'_{bx} + p''_{bx} x_b^m \, . \tag{26}$$

Let $\alpha = x_b^g / x_b$ denote the share of firm g's sales in total sales. Then (26) becomes

$$\Pi_g^{23} = \alpha p'_{bx} \left(\frac{1}{\alpha} + \frac{p''_{bx} x_b}{p'_{bx}} \right)$$

and

$$\Pi_m^{32} = (1 - \alpha) p'_{bx} \left(\frac{1}{1 - \alpha} + \frac{p''_{bx} x_b}{p'_{bx}} \right) . \tag{27}$$

Thus, firm g regards b as a strategic substitute (complement) if the sign of the expression

$$\frac{1}{\alpha} + \frac{p''_{bx} x_b}{p'_{bx}} \tag{28}$$

is positive (negative).[2] The first term in (28) shows how market share affects strategic interaction. This effect is always positive. The second term in (28) measures the curve of the inverse demand function. More precisely, it is the elasticity of inverse marginal demand, p'_{bx}, which shows the influence of demand on strategic interaction. If the demand function, p_{bx}, is concave (linear), then $p''_{bx} x_b / p'_{bx}$ is positive (zero) and we have strategic substitutability. When p_{bx} is convex, strategic complementarity can occur. The larger is firm g's market share (i.e. the higher is α), or the more 'curved' is p_{bx}, the more likely is strategic complementarity.

If one firm (say g) regards b as a strategic complement and the other one (m) regards b as a strategic substitute, then

$$\frac{1}{\alpha} + \frac{p''_{bx} x_b}{p'_{bx}} < 0 \quad \text{and} \quad \frac{1}{1 - \alpha} + \frac{p''_{bx} x_b}{p'_{bx}} > 0 \, . \tag{29}$$

[2] Expression (28) can be helpful in the empirical identification of strategic interaction. We would need to have data on market shares and to estimate the curve of the demand functions.

From (29), we get $\alpha > 1/2$. For this reason, a firm that treats product b as a strategic complement (substitute) must have a higher (lower) market share and therefore an upward (downward) sloping reaction function.[3]

The cross-partial derivative, Π_g^{12}, refers only to firm g's behaviour. It measures the change in the marginal profit from sales of one product when firm g increases sales of a second product and is equal to

$$(p_{ax}^{12}x_a + p_{ax}^{2} + p_{bx}^{21}x_b^g + p_{bx}^{1}) - c_g^{12} . \tag{30}$$

The first term in (30) shows how demand affects Π_g^{12}. If the cross-partial derivatives, p_{ax}^{12} and p_{bx}^{21}, are small, they do not affect the sign of (30). This sign is then determined by the sum $p_{ax}^{2} + p_{bx}^{1}$, which measures the degree of product differentiation. Roughly speaking: the higher is the degree of substitutability (complementarity) of products, the more likely Π_g^{12} is negative (positive). The second term in (30) measures the degree of economies of joint production. We can conclude that economies of joint production and product complementarity require Π_g^{12} to be positive. Conversely, if g has diseconomies of joint production and products are substitutes, then Π_g^{12} is negative. Note that if the goods are independent on the demand side, then $\Pi_g^{12} = -c_g^{12}$ and the whole conception is reduced to (dis)economies of joint production.

The cross-partial derivative, Π_g^{12} can also be interpreted in another way. Taking the partial derivative of the first-order condition (10) with respect to x_b^g and totally differentiating the result, we obtain

$$\frac{dx_a}{dx_b^g} = -\frac{\Pi_g^{12}}{\Pi_g^{11}} . \tag{31}$$

We see that, under the assumption of strict concavity of the profit function, the sign of dx_a/dx_b^g is equal to the sign of Π_g^{12}. Hence, if firm g increases the sales of good b in market x, it will also increase the sales of a in x, when $\Pi_g^{12} > 0$. Conversely, when $\Pi_g^{12} < 0$, it is in g's interest to decrease the sales of good a, whenever the sales of good b have increased.

[3] The same conclusion was obtained by Bulow et al. (1985, p. 500). Another example of a situation where one reaction curve is upward sloping and the other is downward sloping is connected with mixed duopoly: one duopolist is a labour-managed firm which maximizes profit per worker, the second duopolist is a profit maximizer. In this case the labour-managed firm has an upward sloping reaction curve and the profit maximizing firm has a downward sloping reaction curve (see Delbono and Rossini, 1992).

2.2. Second-order and stability conditions

To examine the effect of a change in the exchange rate, we totally differentiate the first-order conditions to obtain a system of six equations which can be written in matrix form:

$$\begin{bmatrix} \Pi_g^{11} & \Pi_g^{12} & \Pi_g^{13} & -c_g^{11} & -c_g^{12} & 0 \\ \Pi_g^{21} & \Pi_g^{22} & \Pi_g^{23} & -c_g^{21} & -c_g^{22} & 0 \\ \Pi_m^{31} & \Pi_m^{32} & \Pi_m^{33} & 0 & 0 & -c_m'' \\ -c_g^{11} & -c_g^{21} & 0 & \Pi_g^{44} & \Pi_g^{45} & \Pi_g^{46} \\ -c_g^{12} & -c_g^{22} & 0 & \Pi_g^{54} & \Pi_g^{55} & \Pi_g^{56} \\ 0 & 0 & -c_m'' & \Pi_m^{64} & \Pi_m^{65} & \Pi_m^{66} \end{bmatrix} \begin{bmatrix} dx_a \\ dx_b^g \\ dx_b^m \\ dy_a \\ dy_b^g \\ dy_b^m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ K \\ L \\ M \\ 0 \end{bmatrix} de , \tag{32}$$

where

$$K = -\frac{\partial \Pi_m^3}{\partial e} = -\left(p_{bx}^2 x_b^m + p_{bx}\right) , \tag{33}$$

$$L = -\frac{\partial \Pi_g^4}{\partial e} = (1/e^2)\left(p_{ay}^1 y_a + p_{ay} + p_{by}^1 y_b^g\right) , \tag{34}$$

$$M = -\frac{\partial \Pi_g^5}{\partial e} = (1/e^2)\left(p_{ay}^2 y_a + p_{by} + p_{by}^2 y_b^g\right) . \tag{35}$$

From (12)–(14) we get

$$K = -(1/e)c_m' < 0 , \tag{36}$$

$$L = (1/e)c_g^1 > 0 , \tag{37}$$

$$M = (1/e)c_g^2 > 0 . \tag{38}$$

We will refer to the matrix of sixth order from Eq. (32) as the matrix $A = [a_{ij}]$. It can be decomposed into four quadratic matrices of the third order:

$$A = \begin{bmatrix} A_1 & A_3 \\ A_3 & A_2 \end{bmatrix} . \tag{39}$$

It is assumed that the matrix A is negative definite, which in particular implies that

- the Nash equilibrium is locally strictly stable,
- the trace of A is negative, which implies that the second-order conditions are satisfied,
- in the absence of market x, market y would be strictly stable and vice versa, hence $\det A_1 < 0$ and $\det A_2 < 0$,

- the market for product b is strictly stable in both countries, i.e.

$$\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} > 0 \text{ and } \begin{vmatrix} a_{55} & a_{56} \\ a_{65} & a_{66} \end{vmatrix} > 0 . \quad (40)$$

3. No strategic interaction

We first investigate the case in which strategic interaction can be neglected, i.e. $\Pi_g^{13} = \Pi_g^{23} = \Pi_m^{31} = \Pi_m^{32} = \Pi_g^{46} = \Pi_g^{56} = \Pi_m^{64} = \Pi_m^{65}$ are equal to zero[4] (or arbitrarily small). The matrix A is then

$$\begin{bmatrix} \Pi_g^{11} & \Pi_g^{12} & 0 & -c_g^{11} & -c_g^{12} & 0 \\ \Pi_g^{21} & \Pi_g^{22} & 0 & -c_g^{21} & -c_g^{22} & 0 \\ 0 & 0 & \Pi_m^{33} & 0 & 0 & -c''_m \\ -c_g^{11} & -c_g^{21} & 0 & \Pi_g^{44} & \Pi_g^{45} & 0 \\ -c_g^{12} & -c_g^{22} & 0 & \Pi_g^{54} & \Pi_g^{55} & 0 \\ 0 & 0 & -c''_m & 0 & 0 & \Pi_m^{66} \end{bmatrix}, \quad (41)$$

and we can decompose the comparative statics system (32) into two systems:

- for firm m:

$$\begin{bmatrix} \Pi_m^{33} & -c''_m \\ -c''_m & \Pi_m^{66} \end{bmatrix} \begin{bmatrix} \mathrm{d}x_b^m \\ \mathrm{d}y_b^m \end{bmatrix} = \begin{bmatrix} K \\ 0 \end{bmatrix} \mathrm{d}e , \quad (42)$$

- and for firm g:

$$\begin{bmatrix} \Pi_g^{11} & \Pi_g^{12} & -c_g^{11} & -c_g^{12} \\ \Pi_g^{21} & \Pi_g^{22} & -c_g^{21} & -c_g^{22} \\ -c_g^{11} & -c_g^{21} & \Pi_g^{44} & \Pi_g^{45} \\ -c_g^{12} & -c_g^{22} & \Pi_g^{54} & \Pi_g^{55} \end{bmatrix} \begin{bmatrix} \mathrm{d}x_a \\ \mathrm{d}x_b^g \\ \mathrm{d}y_a \\ \mathrm{d}y_b^g \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ L \\ M \end{bmatrix} \mathrm{d}e . \quad (43)$$

Both firms are therefore independent. Moreover, they adjust to the

[4] If, moreover, products are independent, then from $\Pi_g^{23} = \Pi_m^{32} = 0$ and (26) we have $x_b^g = x_b^m$. Firms thus share total sales equally and $\alpha = 1/2$. From (27) we get

$$\frac{p''_{bx} x_b}{p'_{bx}} = -2.$$

By solving the above equation we obtain $p_{bx} = A + (B/x_b)$, where A and B are parameters. The assumption about negligible strategic interaction therefore leads to a specific form of the demand function.

exogenous movement of the exchange rate like monopolists.[5] Firm m acts as a monopolist that discriminates across markets. Firm g acts as a monopolist that must consider the effects of joint production when discriminating between markets. We start with firm m.

Proposition 1. If strategic interaction can be neglected, then after a depreciation of the currency of country y firm m will sell more in foreign country x. The change of sales in its domestic market y depends on economies of scale. If firm m has economies (diseconomies) of scale, then it will increase (decrease) sales in country y.

Proof. Solving (42) we get

$$\mathrm{sign}(\mathrm{d}x_b^m/\mathrm{d}e) = \mathrm{sign}\, K\Pi_m^{66} = +$$

and

$$\mathrm{sign}(\mathrm{d}y_b^m/\mathrm{d}e) = \mathrm{sign}\, Kc_m'' = -\,\mathrm{sign}\, c_m''\,. \quad \square$$

The higher marginal revenue in foreign market x pushes firm m to increase sales in country x. That leads to a decrease (increase) in marginal costs if firm m has economies (diseconomies) of scale. In order to equalize marginal revenue with marginal costs in market y, firm m has to increase (decrease) sales in country y.

For firm g we obtain Propositions 2 and 3.

Proposition 2. If strategic interaction can be neglected and products a and b are independent, then after the depreciation of the currency of country y firm g will decrease sales of its two products in both countries if it has economies of scale, i.e. $c_g^{11}, c_g^{22} < 0$ *and economies of joint production, i.e.* $c_g^{12} < 0$.

Proposition 3. If strategic interaction can be neglected and products a and b are differentiated, and $\Pi_g^{12}, \Pi_g^{45} > 0$, *then after a depreciation of the currency in country y:*

(1) *if one of the following three conditions is satisfied,*
  (a) *firm g has only economies of scale, i.e.* $c_g^{11}, c_g^{22} > 0$ *and* $c_g^{12} = 0$, *or*
  (b) *firm g has only economies of joint production, i.e.* $c_g^{11} = c_g^{22} = 0$ *and* $c_g^{12} > 0$, *or*
  (c) *firm g has both economies of scale and economies of joint*

[5] If both firms were monopolists, they would have first-order conditions different from Eqs. (10)–(15), but it is easy to show that the comparative statics would be the same as in (42) and (43).

production, i.e. $c_g^{11}, c_g^{22}, c_g^{12} > 0$, *then firm g will sell less of its two products in both countries*;

(2) *if one of the three following conditions is satisfied*,
  (a) *firm g has only diseconomies of scale, i.e.* $c_g^{11}, c_g^{22} < 0$ *and* $c_g^{12} = 0$, *or*
  (b) *firm g has only diseconomies of joint production, i.e.* $c_g^{11} = c_g^{22} = 0$ *and* $c_g^{12} < 0$, *or*
  (c) *firm g has both diseconomies of scale and diseconomies of joint production, i.e.* $c_g^{11}, c_g^{22}, c_g^{12} < 0$,

then firm g will sell less of its two products in foreign country y and increase sales of its two products in its domestic country x.

The fall in demand in the foreign market y forces firm g to reduce the sales of its two products in market y. That leads to an increase (decrease) in marginal costs if firm g has economies of joint production and/or economies of scale (diseconomies of joint production and/or diseconomies of scale). Firm g then reoptimizes in market x by adjusting marginal revenues to marginal costs and by reducing (raising) sales in market x if marginal costs have increased (decreased).

We can conclude that in the absence of strategic substitutability and strategic complementarity, economies of joint production and/or economies of scale lead to the firm-specific reaction. On the other hand, one can expect the country-specific reaction when the duopolists have diseconomies of scale and/or diseconomies of joint production.

4. The effects of economies of joint production

Consider the situation where the markets are only linked by economies of joint production for firm g i.e. $c_g^{12} \neq 0$. There are no economies of scale if $c_g^{11} = c_g^{22} = c_m'' = 0$. We start with the general situation where products a and b are imperfect substitutes or complements.

Proposition 4. If there are no economies of scale, then after an appreciation in the currency of country x:

(1) *one can expect the firm-specific effect if firm g has economies of joint production, i.e.* $c_g^{12} < 0$, *and both firms regard both goods as strategic substitutes in both markets, i.e.* $\Pi_g^{13}, \Pi_g^{23}, \Pi_m^{31}, \Pi_m^{32}, \Pi_g^{46}, \Pi_g^{56}, \Pi_m^{64}, \Pi_m^{65} < 0$ *and* $\Pi_g^{12}, \Pi_g^{45} > 0$;

(2) *one can expect the country-specific effect if firm g has diseconomies of joint production, i.e.* $c_g^{12} > 0$, *and both firms regard both goods as strategic complements in both markets, i.e.* Π_g^{13}, Π_g^{23}, Π_m^{31}, Π_m^{32}, Π_g^{46}, Π_g^{56}, Π_m^{64}, $\Pi_m^{65} > 0$ *and* Π_g^{12}, $\Pi_g^{45} > 0$.

The explanation is as follows. Firm g reduces sales of its two goods in its foreign market y. Hence, by economies of joint production [see Proposition 3(1)] firm g has an incentive to sell less of good a in its domestic market x (because its sales of b went down in market y), and to reduce sales of good b in market x (because it reduces sales of a abroad).

In domestic market x firm g faces higher sales of good b by firm m. Hence, the strategic substitutability assumption pushes firm g to reduce sales of its two products in market x [remember that strategic substitutability means that the optimal response to increased sales by the competitor is to decrease sales; see Eq. (21)].

Economies of joint production and strategic substitutability both push firm g to reduce sales in its domestic market.

When firm g is confronted with diseconomies of scale and strategic complementarity [see Proposition 3(2)], this combination causes firm g to increase sales of its two products in domestic market x. In this case, however, the condition $\Pi_g^{12} > 0$ means in particular, by (30), that

$$\Pi_g^{12} = (p_{ax}^{12} x_a + p_{ax}^2 + p_{bx}^{21} x_b^g + p_{bx}^1) - c_g^{12} > 0 \,, \tag{44}$$

and analogously for $\Pi_g^{45} > 0$.

The expression

$$p_{ax}^{12} x_a + p_{ax}^2 + p_{bx}^{21} x_b^g + p_{bx}^1$$

which, roughly speaking, measures the degree of product complementarity, must not only be positive but also greater than $-c_g^{12}$. Thus, intuitively speaking, point (2) is valid only when the degree of product complementarity is high.[6]

Suppose now that goods a and b are independent on the demand side,[7] so that the relation between markets is established only through economies of joint production. In this case

$$A_1 = \begin{bmatrix} \Pi_g^{11} & -c_g^{12} & 0 \\ -c_g^{12} & \Pi_g^{22} & \Pi_g^{23} \\ 0 & \Pi_m^{32} & \Pi_m^{33} \end{bmatrix}, \qquad A_2 = \begin{bmatrix} \Pi_g^{44} & -c_g^{12} & 0 \\ -c_g^{12} & \Pi_g^{55} & \Pi_g^{56} \\ 0 & \Pi_m^{65} & \Pi_m^{66} \end{bmatrix}. \tag{45}$$

[6] Note that in the linear framework $\Pi_g^{12} > 0$ and $\Pi_g^{45} > 0$ if and only if goods are complements on the demand side.

[7] Note that if, in addition, the cost functions were linear, we would have a standard monopolist selling product a in markets x and y on the one hand, and duopolists selling a homogeneous product b in each other's markets on the other hand. The latter case in handled in Hens et al. (1991) and there is no point repeating their analysis here, except to recall that they show strategic substitutability to imply the firm-specific effect.

Proposition 5. If products are independent, there are no economies of scale and firm g exhibits economies of joint production, i.e. $c_g^{12} < 0$, *then after an appreciation of the currency in country x we have the firm-specific effect if firm g regards good b as a strategic substitute in market x, i.e.* $\Pi_g^{23} < 0$ *and firm m regards good b as a strategic substitute in both markets, i.e.* $\Pi_m^{32}, \Pi_m^{65} < 0$. *Moreover, the total sales of good b decrease in country y.*

Therefore the price of product a increases in both countries and the price of product b increases in country y.

Proposition 5 can be considered as a special case of Proposition 4 (see also Proposition 2). In the absence of product differentiation, strategic substitutability of b and economies of joint production for g lead to the firm-specific effect.

Note that if the products are independent, we have from (44):

$$\Pi_g^{12} = \Pi_g^{45} = -c_g^{12},$$

which means that $\Pi_g^{12} > 0$ if and only if firm g has economies of joint production. Therefore we can say nothing in this case about the effects of diseconomies of joint production.

5. The effects of economies of scale

Suppose $c_g^{ij} = 0$ for $i \neq j$ so that the matrix A_3 is diagonal. Markets x and y are linked through economies of scale only. However, markets for goods a and b are separated. Firm g can therefore be considered as a monopolist which is selling good a in two markets and price discriminates across markets.

Proposition 6. If products are independent and there are no economies of joint production, then after a change in the exchange rate:

(1) *in the case of economies of scale, i.e.* $c_g^{11}, c_g^{22}, c_m'' < 0$, *one can expect the firm-specific effect if b is regarded as a strategic substitute by both players in both countries, i.e.* $\Pi_g^{23}, \Pi_g^{56}, \Pi_m^{32}, \Pi_m^{65} < 0$. *Therefore prices of product a increase in both countries.*

(2) *in the case of diseconomies of scale, i.e.* $c_g^{11}, c_g^{22}, c_m'' > 0$, *we can expect the country-specific effect if b is regarded as a strategic complement by both players, i.e.* $\Pi_g^{23}, \Pi_g^{56}, \Pi_m^{32}, \Pi_m^{65} > 0$. *Therefore both prices go down in the country whose currency appreciates, and both prices go up in the other country.*

The intuition is straightforward. Firm m faces lower sales of good b by firm g in market y. Hence, by strategic substitutability, firm m has an incentive to increase sales in the home market. Moreover, firm m sells more abroad. This decreases its marginal cost and, because of economies of scale, firm m raises sales at home. As a consequence, strategic substitutability combined with economies of scale give firm m an incentive to increase sales in its domestic market.

When firm m is confronted with diseconomies of scale and strategic complementarity, this combination causes firm m to reduce sales in its domestic market.

For firm g, both strategic substitutability and economies of scale push firm g to reduce sales of good b in its domestic market x. The combination of strategic complementarity and diseconomies of scale works in the opposite direction.

Proposition 6 is a straightforward generalization to a situation with two independent commodities. See Proposition 5 (which considers markets with one commodity) in Kirman and Phlips (1992). Notice that diseconomies of scale make sure that both prices move in the normal direction (down in the country that appreciates, up in the other country) when coupled with strategic complementarity for b. In the economies of scale case, all we are able to say about prices is that, since g will sell less of a in both countries, its price will go up in both countries too. This is a surprising result from the point of view of the standard literature on exchange rate pass-through. Yet, it is easily understood, since g has the monopoly of commodity a when a is independent of b from the consumer's point of view.

6. Conclusions

In this paper the effects of a change in the exchange rate on duopolistic behaviour are examined in the framework of a two-commodity model with joint production.

Our six propositions together imply that strategic substitutability or strategic complementarity are the dominating forces whether the two commodities are substitutes, complements or independent from the consumer's point of view.

From the point of view of policy-makers, a devaluation should improve domestic firm competitiveness in international markets and not lead to price increases. Policy-makers would like the firm-specific effect to occur, not the country-specific effect, so that domestic firms sell more at home and abroad and domestic prices go down. Hence it is important to know which factors induce the firm-specific effect and which factors lead to the country-specific effect.

From the results obtained we can conclude that the existence of strategic substitutability, economies of scale and/or economies of joint production lead to the firm-specific effect.

If products are strategic complements and there are diseconomies of scale and/or diseconomies of joint production, then we can expect a country-specific effect.

There are still some open questions in this model. We can ask about the joint effect of factors which work in opposite directions. For example: What is the effect of economies of scale *and* strategic complementarity? There is also the very interesting question of how a devaluation affects duopolists with objective functions other than profit maximization.

Acknowledgements

We wish to thank Berthold Herrendorf, Curtis Eberwein, Robert Waldmann and two referees for helpful comments. Of course, we are responsible for all views expressed and any errors. The final version of the paper was written while Baniak was a fellow of the International Centre for Economic Research, Torino, Italy.

References

Bulow, J.I., J. Geanakoplos and P. Klemperer, 1985, Multimarket oligopoly: Strategic substitutes and complements, Journal of Political Economy 93, 488–511.

Delbono, F. and G. Rossini, 1992, Competition policy vs horizontal merger with public, entrepreneurial, and labor-managed firms, Journal of Comparative Economics 16, 226–240.

Hens, T., A. Kirman and L. Phlips, 1991, Exchange rates and oligopoly, EUI Working Paper, ECO no. 91/42, European University Institute, Florence.

Kirman, A. and L. Phlips, 1992, Exchange rate pass-through and market structure, EUI Working Paper, ECO no. 92/83, European University Institute, Florence.

Martin, S. and L. Phlips, 1994, Product differentiation, market structure and exchange rate passthrough, in: A. von Witteloostuÿn, ed., Market evolution: competition and cooperation across markets and over time (Kluwer Academic Publishers, Dordrecht, 1994).